NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3751

A STUDY OF THE EFFICIENCY OF HIGH-STRENGTH,

STEEL, CELLULAR-CORE SANDWICH PLATES IN COMPRESSION

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NACA

Washington

September 1956

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SUMMARY

The structural efficiency of high-strength, stainless-steel, cellular-core sandwich plates is investigated. Efficiency curves are presented for sandwich plates of various proportions subjected to compressive end loads for temperatures of 80° F and 600° F. Optimum proportions of sandwich plates for any value of the compressive loading intensity can be determined from the curves. A comparison is made between the efficiency of optimum-proportioned steel sandwich plates and solid plates of high-strength steel and aluminum and titanium alloys at the two temperatures.

INTRODUCTION

Composite or, more familiarly, sandwich plates have long appealed to the designer because, in a given design application, the load-carrying material can be distributed to provide optimum combinations of stiffness and strength for a given weight. Because of this inherent advantage, a great deal of effort has been expended in recent years to develop suitable sandwich materials and to provide design data for the various forms of sandwich materials. (See, for example, the bibliography in ref. 1.)

In spite of the large amount of development work, the use of high-strength sandwich material for primary structural applications has not been fully exploited, principally because of fabrication difficulties such as questionable or unreliable bonding of the sandwich elements to each other. Recent developments, however, in the fabrication of sandwich plates by means of high-temperature metallurgical joining techniques (see, for example, refs. 2 to 6) have indicated bonding processes which have the potential to provide continuous, high-strength, reliable joining of components. One implication of this development is that, by high-temperature brazing or welding, stainless-steel sandwich plates may be fabricated which can be used at the higher temperatures associated with high-speed flight where the lighter metals are no longer usable. For some loading conditions

a steel sandwich plate, designed for efficient use at elevated tempera-
tures, may actually weigh no more than a solid plate of the lighter
alloys which are designed to carry the same loads at room temperature.

This paper presents the results of calculations for the structural
efficiency in compression for the cellular-core type of sandwich plate.
(See fig. 1.) The study was made for plates fabricated from a typical
high-strength stainless steel, and the sandwich proportions necessary
for optimum efficiency for various loading intensities and two tempera-
tures were found. The results are presented in the form of plate effi-
ciency curves, and these curves are compared with similar curves for
solid plates of steel and aluminum and titanium alloys at room tempera-
ture (80° F) and 600° F.

SYMBOLS

b	width of plate element, in.
D	plate flexural stiffness, kip-in.
D_Q	shear stiffness, kips/in.
E	modulus of elasticity, ksi
E_f	modulus of elasticity of facing material, ksi
G_c	modulus of rigidity of core material, ksi
h	overall thickness of sandwich plate, $h_c + 2t_f$, in.
h_c	thickness of core, in. (see fig. 1)
k	nondimensional compressive buckling coefficient
P_i	compressive load carried by plate per unit width, kips/in.
r	shear-stiffness parameter
t_f	thickness of facing material, in.
$\bar{t}$	cross-sectional area of sandwich plate per unit width, expressed as an equivalent thickness, in.

η   plasticity reduction factor

μ   Poisson's ratio

ρ   density of solid plates, lb/cu in.

ρ_c effective core density, lb/cu in.

ρ_f facing-material density, lb/cu in.

σ   compressive stress in the facings, ksi

σ_{cy} material compressive yield stress, ksi

Subscript:

el   elastic

METHOD OF ANALYSIS

The efficiency analysis of this paper is based primarily on existing theories for the compressive buckling behavior of sandwich plates which were formulated on the basis of assumed physical characteristics of the sandwiches. These assumptions are retained in the present analysis. The assumptions are discussed first, and then the efficiency equation is derived.

Assumptions

The following characteristics are assumed for the sandwich plates analyzed herein:

(1) The facing sheets are of equal thickness, are plane, are rigidly bonded to the core, and have infinite shear stiffness normal to the plane of the plate.

(2) The applied compressive loads are carried only by the facing sheets.

(3) The facing sheets are continuously and uniformly supported by the core; that is, the dimensions of a core cell are small enough to prevent local buckling of the facing sheets within a cell boundary before the sandwich plate buckles as a unit.

(4) The core is a homogeneous, isotropic medium of finite shearing stiffness and has infinite extensional stiffness in the direction normal to the facing sheets.

(5) The maximum edge compressive loading that can be sustained is the simply supported buckling load for the sandwich plate behaving as an integral unit.

The assumption that the core carries no axial load is not precisely representative of cellular-core sandwich construction. In actuality, the core is compressed to the same axial deformation as the facing sheets which could result in local buckling of the cell walls. Such buckling reduces the effective shear modulus of the core and, hence, may reduce the plate strength for some sandwich proportions. The sandwiches most likely to be affected are those with a very low density core in which the ability to carry compressive load is already strongly influenced by low shear stiffness.

The possibility of sandwich failure of the type commonly called "wrinkling" is excluded in the analysis. In this type of failure, the facing sheets buckle as plates supported by an elastic core, which is deformable in a direction normal to the facings. However, from the buckling theory of reference 7 (which includes both symmetric and anti-symmetric modes of wrinkling), it can be shown that, for the range of sandwich proportions covered by the present investigation, wrinkling instability in the facings will always occur at higher stresses than the buckling stress of the sandwich plate as a unit.

Initial imperfections in the facing sheets which resemble local buckles confined within the cell boundaries may occur in the manufacture of cellular-core sandwich. These imperfections are indicative of cell dimensions which are probably too large for the facing plate thickness and have the effect of reducing the effective modulus of the facing plate material. Imperfections of this type can seriously reduce the flexural stiffness of the sandwich; however, their effect is not included in the present analysis.

Derivation of Efficiency Equations

The isotropic plate buckling equation

$$P_i = \frac{k\pi^2\eta D}{b^2} \tag{1}$$

subject to the assumptions listed previously was used as a failure criterion to calculate the compressive efficiency of sandwich plates of various

proportions. The effect of shearing deformation is included in the buckling load coefficient k. The expressions for k, derived in reference 8, applicable to plates infinitely long in the loading direction were used:

$$k = \frac{4}{(1 + r)^2} \qquad (r \leqq 1) \qquad (2a)$$

$$k = \frac{1}{r} \qquad (r \geqq 1) \qquad (2b)$$

The nondimensional shear-stiffness parameter r is defined as

$$r = \frac{\pi^2 D}{b^2 D_Q} \qquad (3)$$

As used in reference 8, the expression for r considers the thickness of the facings to be negligible in comparison with the overall thickness of sandwich. For the type of sandwich plates considered in this paper, considerable errors will result (particularly at low values of $h/2t_f$ and ρ_c/ρ_f) unless the facing thickness is included in the expression for the flexural and shear stiffnesses D and D_Q, respectively. The flexural stiffness D, expressed in terms of the sandwich-plate dimensions, is thus

$$D = \frac{2E_f}{1 - \mu^2} t_f{}^3 \left[\left(\frac{h}{2t_f} - \frac{1}{2} \right)^2 + \frac{1}{12} \right] \qquad (4)$$

and the shear stiffness D_Q may be defined as (see eq. (5), ref. 9)

$$D_Q = \frac{2G_c t_f \left(\frac{h}{2t_f} - \frac{1}{2} \right)^2}{\frac{h}{2t_f} - 1} \qquad (5)$$

The results of reference 10 indicate that a conservative value for the shear modulus of the core G_c can be determined from a consideration of the volume of core material oriented in the plane of the shear forces.

For a core as shown in figure 1, one-half of the shear stiffness of the core material can be considered effective in any plane normal to the faces. For core material which has nominally the same ratio of Young's modulus to density as the facing material, G_c may be written as

$$G_c = \frac{1}{4} \frac{\rho_c}{\rho_f} \frac{E_f}{1 + \mu} \tag{6}$$

This equation also implies that the material in the welded or brazed joint carries shear as effectively as the core.

When the cross-sectional area of the plate per unit width is defined as

$$\bar{t} = 2t_f + \frac{\rho_c}{\rho_f} h_c \tag{7a}$$

or, in nondimensional form,

$$\frac{\bar{t}}{b} = \frac{2t_f}{b}\left[1 + \frac{\rho_c}{\rho_f}\left(\frac{h}{2t_f} - 1\right)\right] \tag{7b}$$

an expression for r can be written in terms of sandwich-plate parameters $\bar{t}/b$, ρ_c/ρ_f, and $h/2t_f$ which are nondimensional. When equations (4), (5), (6), and (7b) are combined and substituted into equation (3), the following expression for r is obtained:

$$r = \pi^2 \frac{1 + \mu}{1 - \mu^2} \frac{\rho_f}{\rho_c}\left(\frac{\bar{t}}{b}\right)^2 \frac{\left[\left(\frac{h}{2t_f} - \frac{1}{2}\right)^2 + \frac{1}{12}\right]\left(\frac{h}{2t_f} - 1\right)}{\left[1 + \frac{\rho_c}{\rho_f}\left(\frac{h}{2t_f} - 1\right)\right]^2 \left(\frac{h}{2t_f} - \frac{1}{2}\right)^2} \tag{8}$$

Values of r from equation (8) are used to calculate elastic values of the buckling load coefficient k (eqs. (2)) corrected for shearing deformations in the core of the sandwich plate.

The maximum compressive load that can be carried by a sandwich plate of width b and cross-sectional area $\bar{t}$ is obtained by combining equations (1), (4), and (7b) to give

$$\frac{P_i}{b} = \frac{k\pi^2\eta E_f}{4(1 - \mu^2)}\left(\frac{\bar{t}}{b}\right)^3 \frac{\left(\frac{h}{2t_f} - \frac{1}{2}\right)^2 + \frac{1}{12}}{1 + \frac{\rho_c}{\rho_f}\left(\frac{h}{2t_f} - 1\right)} \tag{9}$$

In this equation the allowable loading P_i has been divided by the plate width to form the structural-index parameter P_i/b which is a function of $\bar{t}/b$ (a measure of the weight of the sandwich plate), the sandwich proportions, and the material properties.

For solutions of equation (9) which involve stresses higher than the elastic limit of the facing material, an assumed reduction factor η, corresponding to that used for inelastic buckling stresses of solid plates, is used. An elastic value of P_i/b is first calculated from equation (9) (by assuming a value of unity for η) and the corresponding elastic buckling stress is found from the relation

$$\sigma_{el} = \left(\frac{P_i}{b}\right)_{el} \frac{b}{\bar{t}}\left[1 + \frac{\rho_c}{\rho_f}\left(\frac{h}{2t_f} - 1\right)\right] \tag{10}$$

For the present analysis, the reduction factor η was determined from reference 11 and plotted against the corresponding elastic stress. From this plot, a value of η was obtained for the facing material, and the appropriate value of the structural-index parameter was determined from

$$\frac{P_i}{b} = \eta\left(\frac{P_i}{b}\right)_{el} \tag{11}$$

RESULTS AND DISCUSSION

Efficiency Curves

The plate compressive efficiency of cellular-core sandwich plates has been calculated from equation (9) and is shown in figures 2 and 3 for room temperature (80° F) and 600° F, respectively. The material properties used in the calculations are typical of high-strength stainless steel. (See table I.) The efficiency curves give the cross-sectional area $\bar{t}$ required to carry the compressive loading intensity P_i over any arbitrary width b for the specified ratios of core density to facing density ρ_c/ρ_f and sandwich thickness to total facing thickness $h/2t_f$.

Along any curve, the stress in the facings at buckling (assumed to be failure) varies from zero at the origin to a maximum value which was limited in these calculations to the 0.2-percent-offset compressive yield stress for the material. This maximum-stress restriction results in a straight portion at the end of each curve. The stress associated with any point on the curves is given by

$$\sigma = \frac{P_i}{b}\frac{b}{\bar{t}}\left[1 + \frac{\rho_c}{\rho_f}\left(\frac{h}{2t_f} - 1\right)\right] \tag{12}$$

An optimum ratio of sandwich thickness to total facing thickness for minimum weight at each value of the structural index can be determined from the efficiency curves. Along any curve for a particular sandwich-plate proportion, there is only one value of loading intensity for which this proportion is optimum. The dashed envelope curve drawn tangent to the individual curves at these loading intensities defines the optimum proportions of sandwich plates. The failing stress in the facings associated with any point along the envelope curve is slightly below the compressive yield stress for the facing material. Thus, the design of optimum sandwich plates resolves into the selection of sufficient core thickness and density to attain the compressive yield stress in the facing sheets. This feature is characteristic of the curves for both temperatures; hence, the change in weight associated with a change in the temperature of sandwich plates of optimum design is essentially inversely proportional to the change in material yield stress.

The effect of core shear stiffness on plate compressive efficiency is illustrated by the trend of the curves for sandwich proportions having low values of $h/2t_f$ and ρ_c/ρ_f. Because of inadequate core shear

stiffness, the compressive yield stress cannot be achieved in the facing sheets and the curves lie above the envelope of optimum proportions at all values of loading intensity.

The influence of core density on structural efficiency of high-strength, steel, cellular-core sandwich plates is shown in figures 4(a) and 4(b) where the lower envelopes to the curves of figures 2 and 3 have been plotted. In general, the compressive efficiency increases with decreasing core density, but the gain is slight below values of ρ_c/ρ_f equal to approximately 0.02 at low loading intensities and below values of ρ_c/ρ_f equal to approximately 0.05 at the high loading intensities. It is apparent that the expected savings in weight associated with very low density cores is nullified by core shear flexibility. In actuality, production of satisfactory panels with very low density cores becomes increasingly difficult.

The dashed curves in figure 4 represent idealized sandwich plates in which the facings are assumed to develop the material compressive yield stress with a core of zero weight. For any value of the loading index, the difference in ordinates between the solid- and dashed-line curves corresponds to the weight of material in the core. It is evident that for sandwich plates of optimum proportions, the weight of the core material relative to the weight of the complete sandwich plate decreases with increasing loading intensity. Except for low values of P_i/b, only a small percentage of the weight of the sandwich plate of optimum proportions is supplied by the core material. This fact is exemplified by the decreasing values of $h/2t_f$ for optimum design as P_i/b increases.

Comparison With Solid Plates

A comparison of the load-carrying efficiency of optimum-proportioned steel sandwich plates with solid plates of other materials is given in figure 5. Figure 5(a) gives the comparison at a room temperature of 80° F between steel sandwich plates and solid plates of a high-strength aluminum alloy, of a high-strength titanium alloy, and of the same steel assumed for the sandwich plates. (See table I for summary of properties of materials.) The sandwich-plate curves are based on a buckling criterion, whereas the solid-plate curves are determined from the maximum strength of supported plates (as shown in ref. 12). In order that the weights of the different plates at the same loading intensity may be compared directly, the ordinate of figure 5 includes the effect of the different material densities. From these comparisons, it is evident that it is possible to design steel sandwich plates which are considerably more efficient than solid plates of titanium alloy over the entire range considered and which are more efficient than the aluminum alloy over the

lower portion of the loading range. Similar results are evident in the
comparisons between the steel sandwich plates and solid plates of tita-
nium alloy and solid plates of the steel at a temperature of 600° F.
(See fig. 5(b).)

Because of the relatively small drop in the material properties of
steel at 600° F, an optimum sandwich plate of this material at this
temperature is more efficient than solid plates of the titanium alloy
at room temperature over a large range of loading intensity and is com-
parable in efficiency to plates of aluminum alloy at room temperature
for low loading intensities. These comparisons indicate that the den-
sity disadvantage of a temperature-resistant material such as stainless
steel may be overcome by fabrication into a sandwich.

CONCLUSIONS

From the analysis and curves presented for the efficiency of high-
strength, steel, cellular-core sandwich plates subjected to compressive
end loads, the following are evident:

1. Sandwich plates of optimum design require the use of the minimum
core thickness and density necessary to attain approximately the yield
stress in the facing sheets at the design value of the structural index.

2. The weight of the core material relative to the weight of the
complete sandwich plate decreases with increasing loading intensity.
Except for low values of compressive loading intensity, the core thick-
ness in steel sandwich plates of efficient design is small and conse-
quently the weight of the core material is only a small percentage of
the weight of the sandwich plate.

3. The compressive efficiency of a steel sandwich plate first
increases with decreasing core density, but as the core density is
reduced below about 2 percent of that of the facing material the shear
stiffness of the core becomes an important factor and finally results
in a decrease in efficiency at higher values of loading. At low loading
intensities, there is little gain in efficiency by reducing the core
density below approximately 2 percent of that of the facing material.
At high loading intensities, core densities approximately 5 percent of
that of the facing material are indicated to be near optimum.

4. At room temperature, steel sandwich plates can be built which
are more efficient than solid plates of titanium alloy over the entire
loading range considered and which are more efficient than solid plates
of aluminum alloy over the lower part of the loading range. At 600° F,
and over the entire loading range considered, steel sandwich plates are

more efficient than solid plates of titanium alloy. For low values of loading intensity, the efficiency of steel sandwich plates at a temperature of 600° F is greater than the efficiency of solid plates of the aluminum or titanium alloys at 80° F.

5. For efficiently designed sandwich plates, the changes in weight associated with a change in temperature are closely related to the change in material yield stress with temperature.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., May 31, 1956.

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TABLE I

PROPERTIES OF MATERIALS

Material	Density, ρ, lb/cu in.	80° F		600° F	
		Young's modulus, E, ksi	Compressive yield stress, σ_{cy}, ksi	Young's modulus, E, ksi	Compressive yield stress, σ_{cy}, ksi
Steel	0.30	30.0×10^3	180	27.5×10^3	150
Aluminum alloy	.10	10.5	72.5	----	---
Titanium alloy	.17	16.1	140	12.7	81
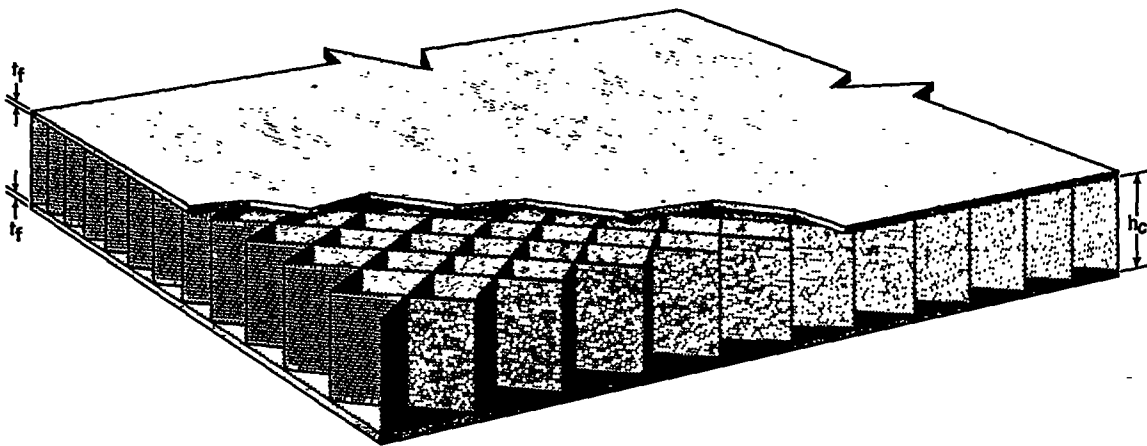
Figure 1.- Section of typical cellular-core sandwich plate.
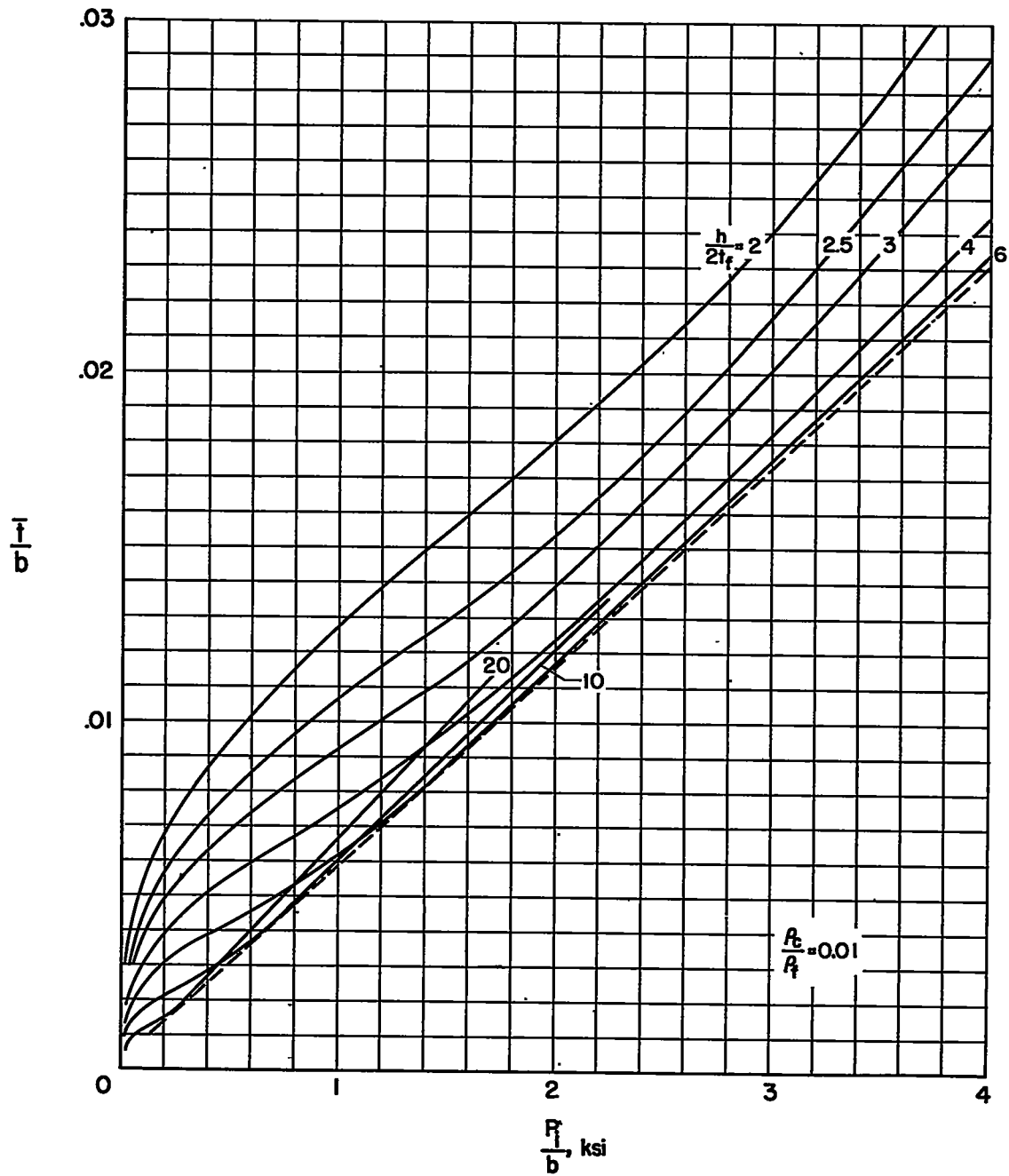
Figure 2.- Structural efficiency of high-strength, steel, cellular-core sandwich plates at 80° F.
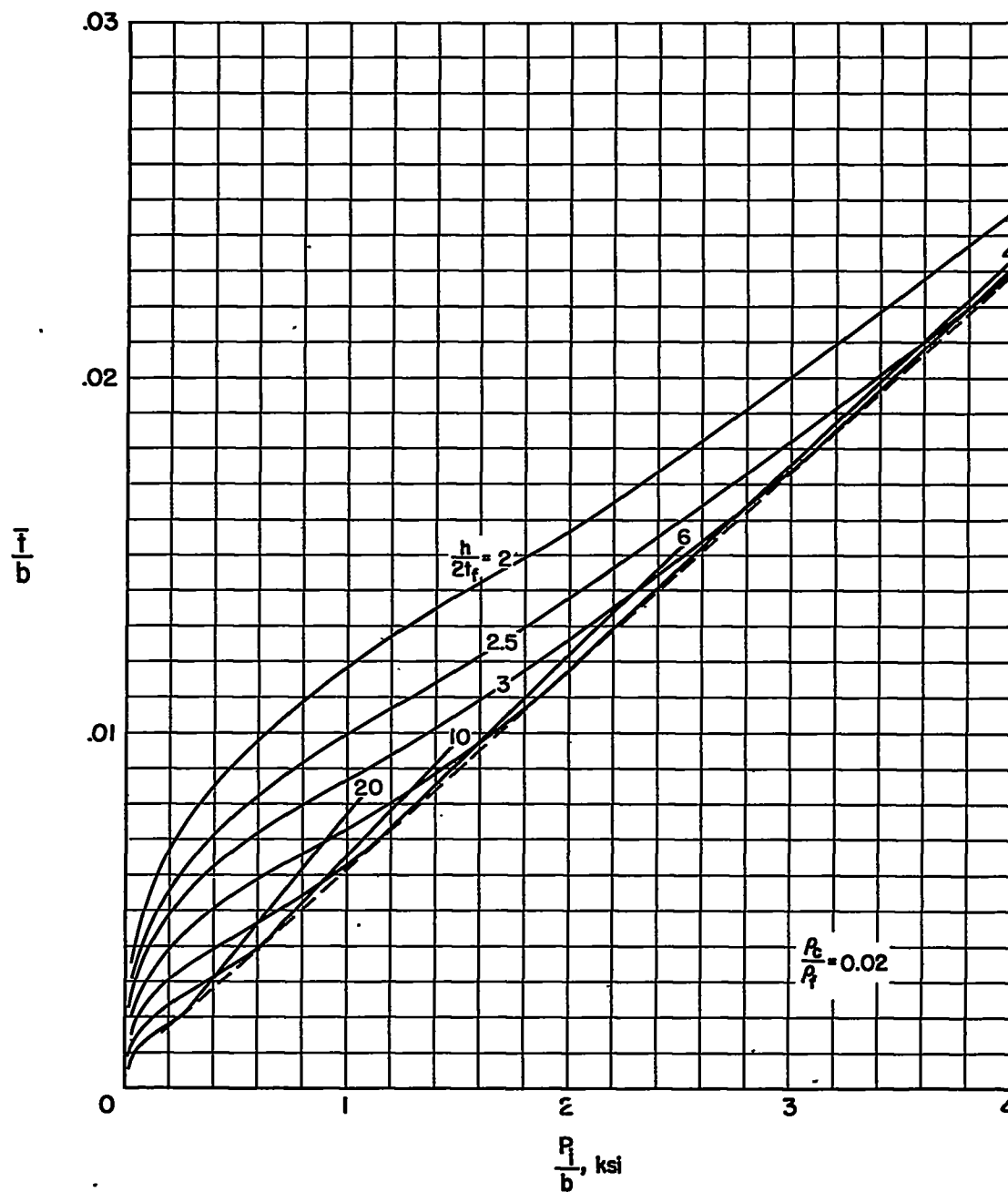
Figure 2.- Continued.
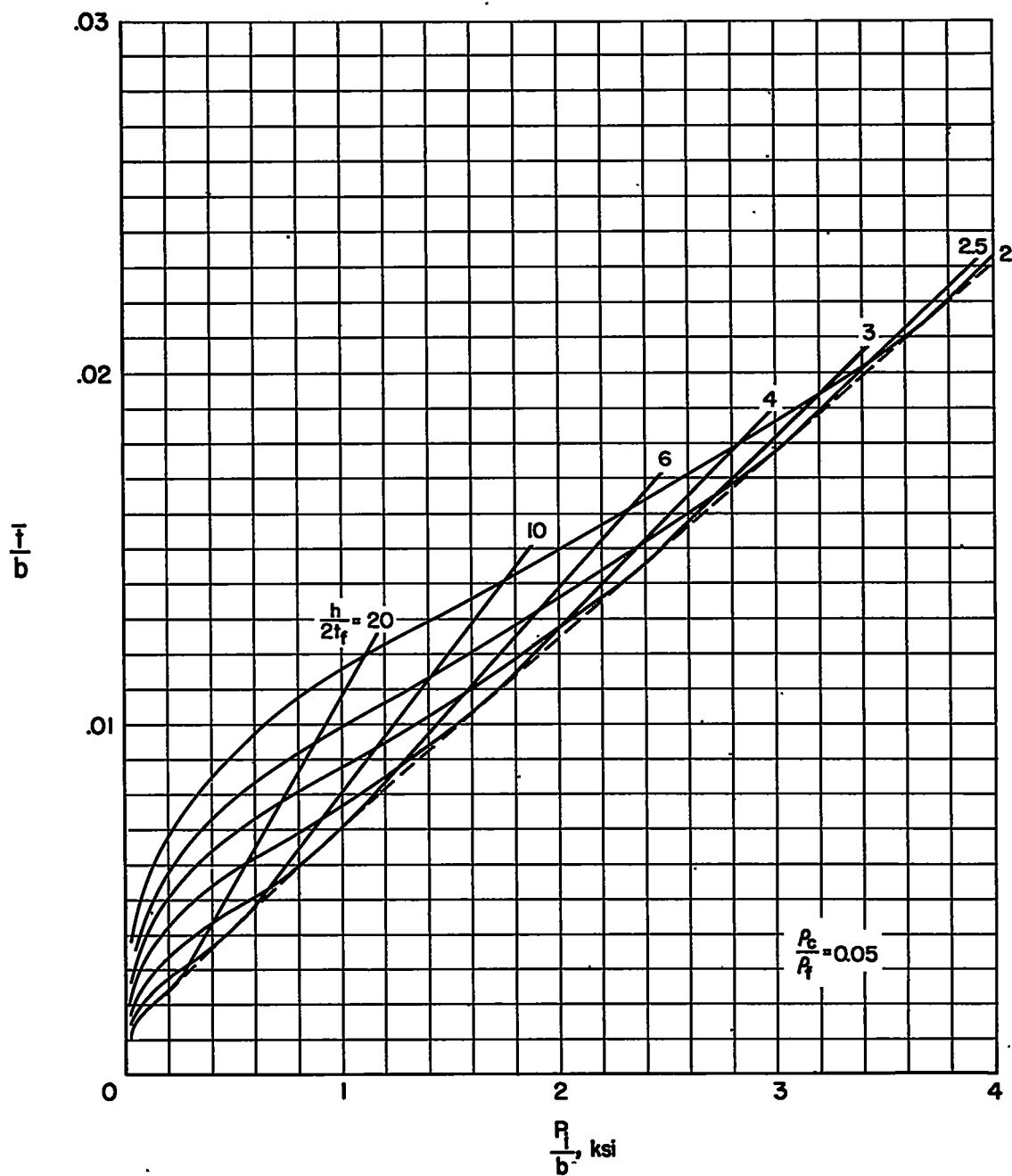
Figure 2.- Continued.
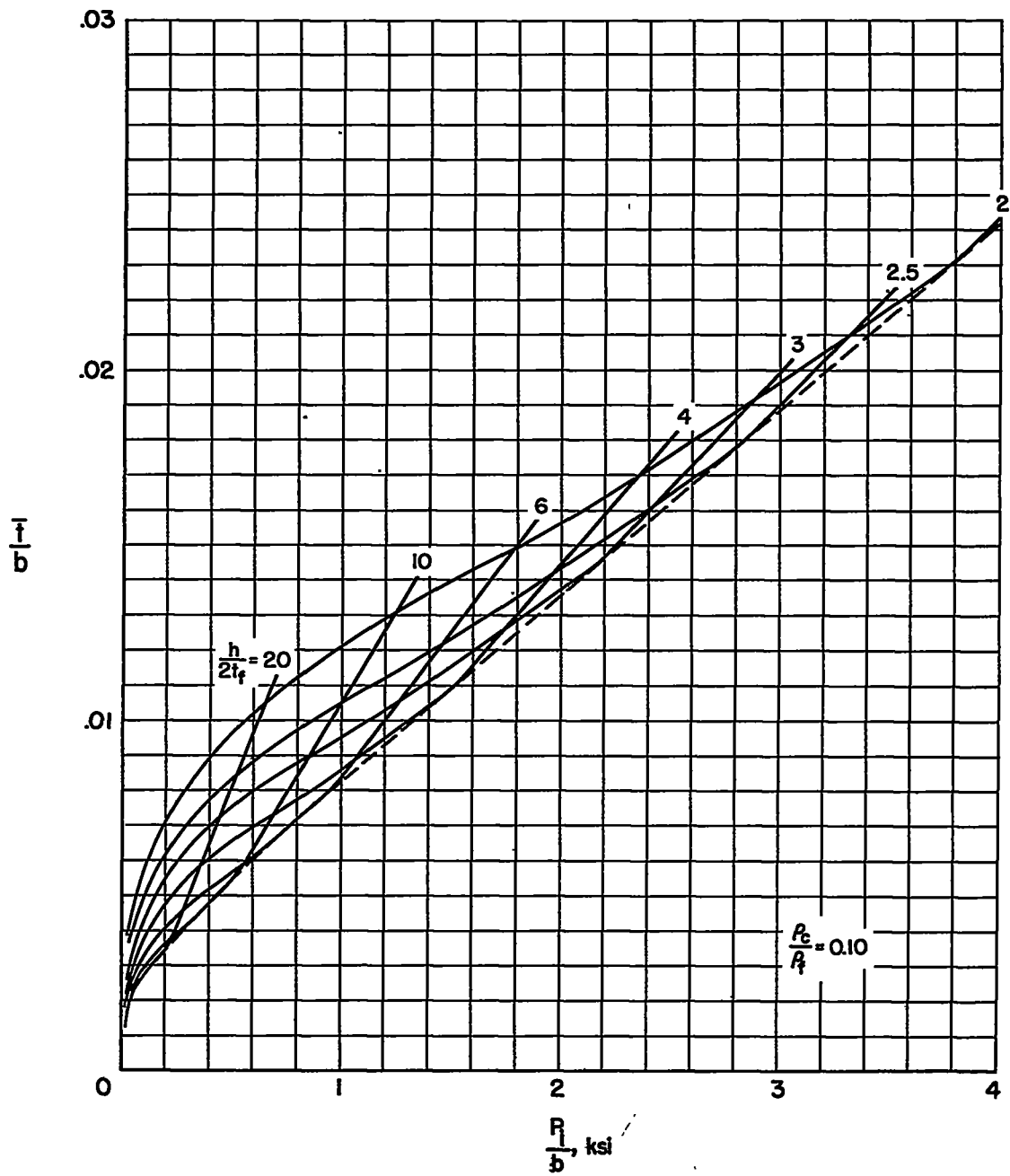
Figure 2.- Concluded.
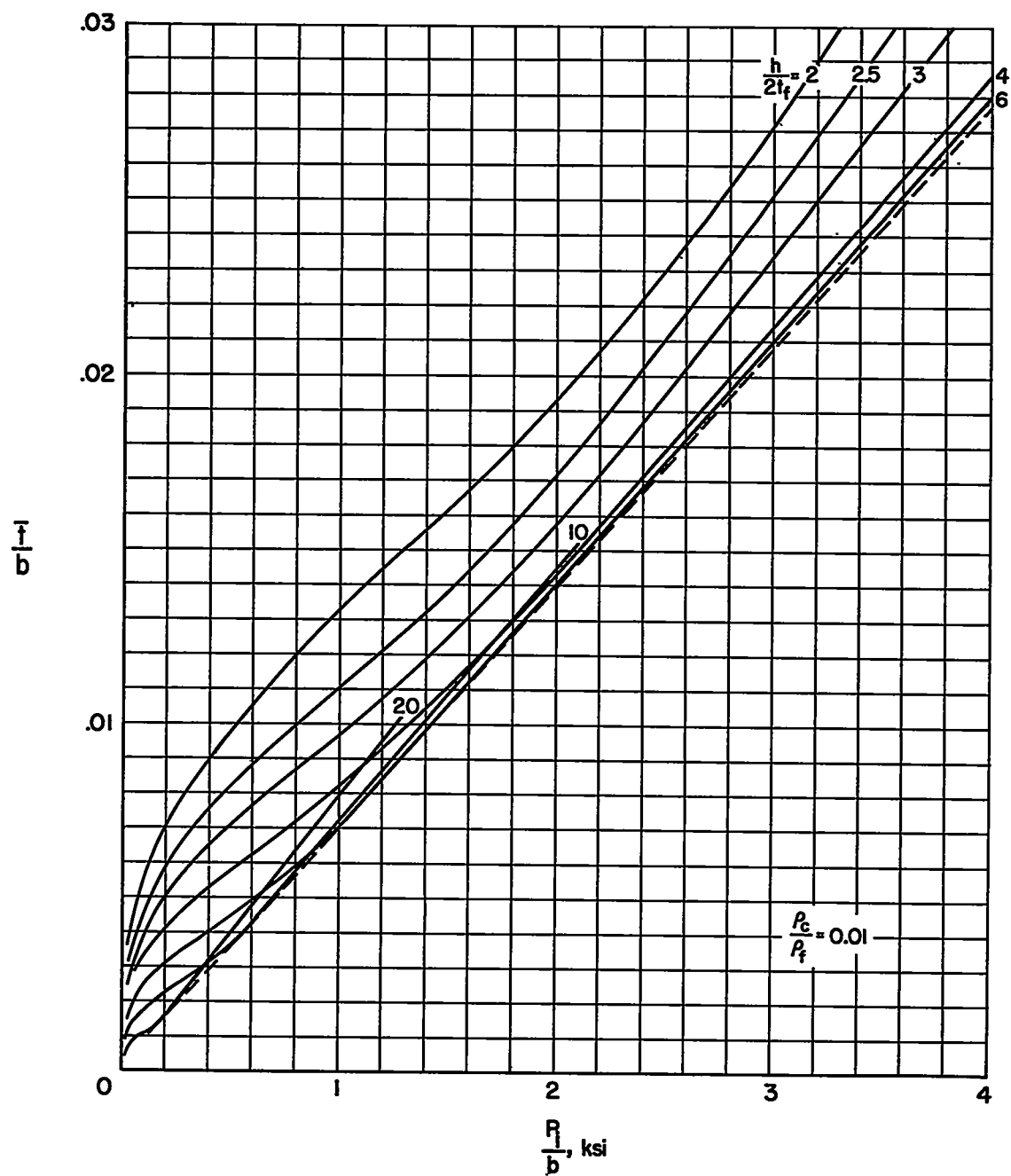
Figure 3.- Structural efficiency of high-strength, steel, cellular-core sandwich plates at 600° F.
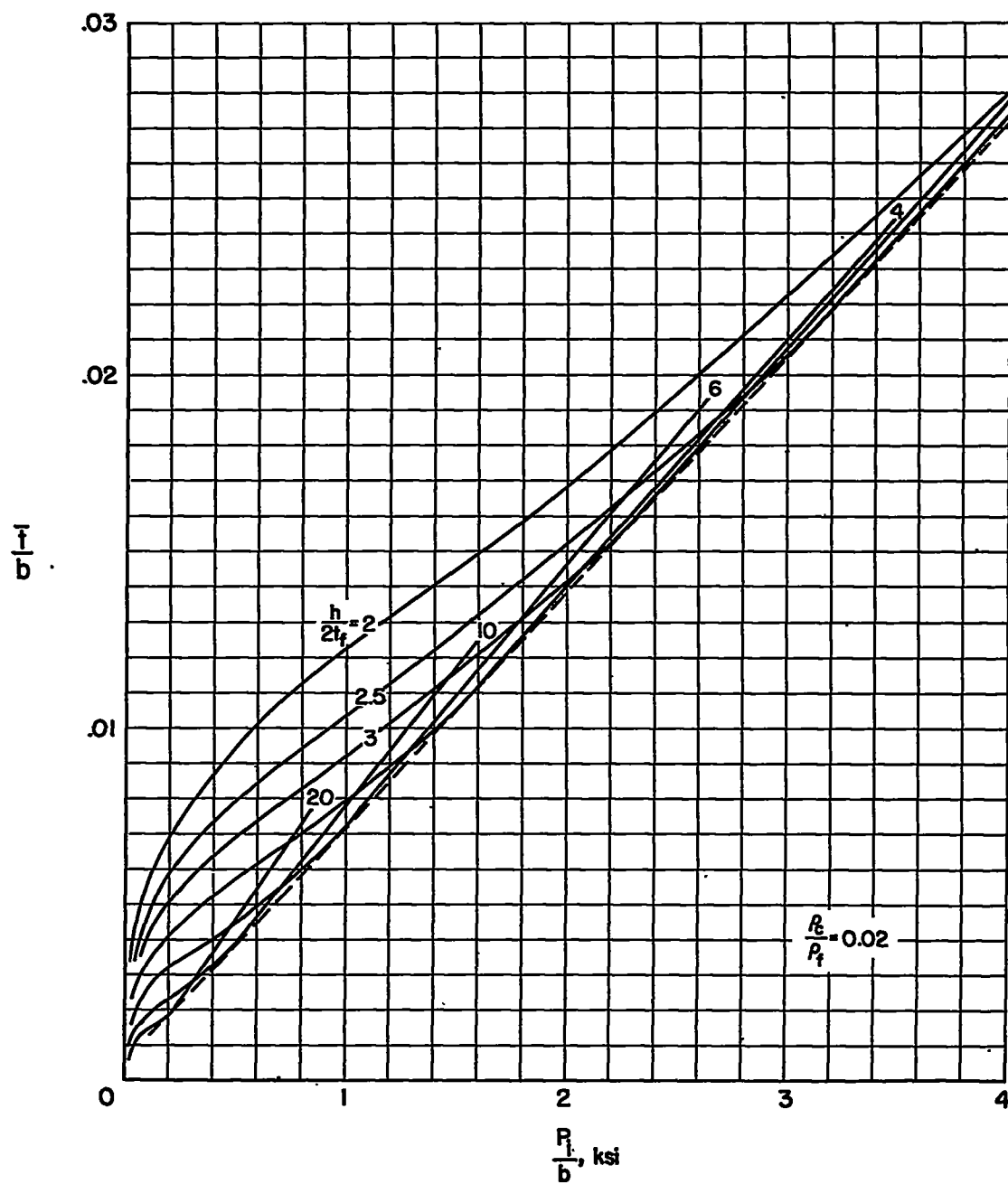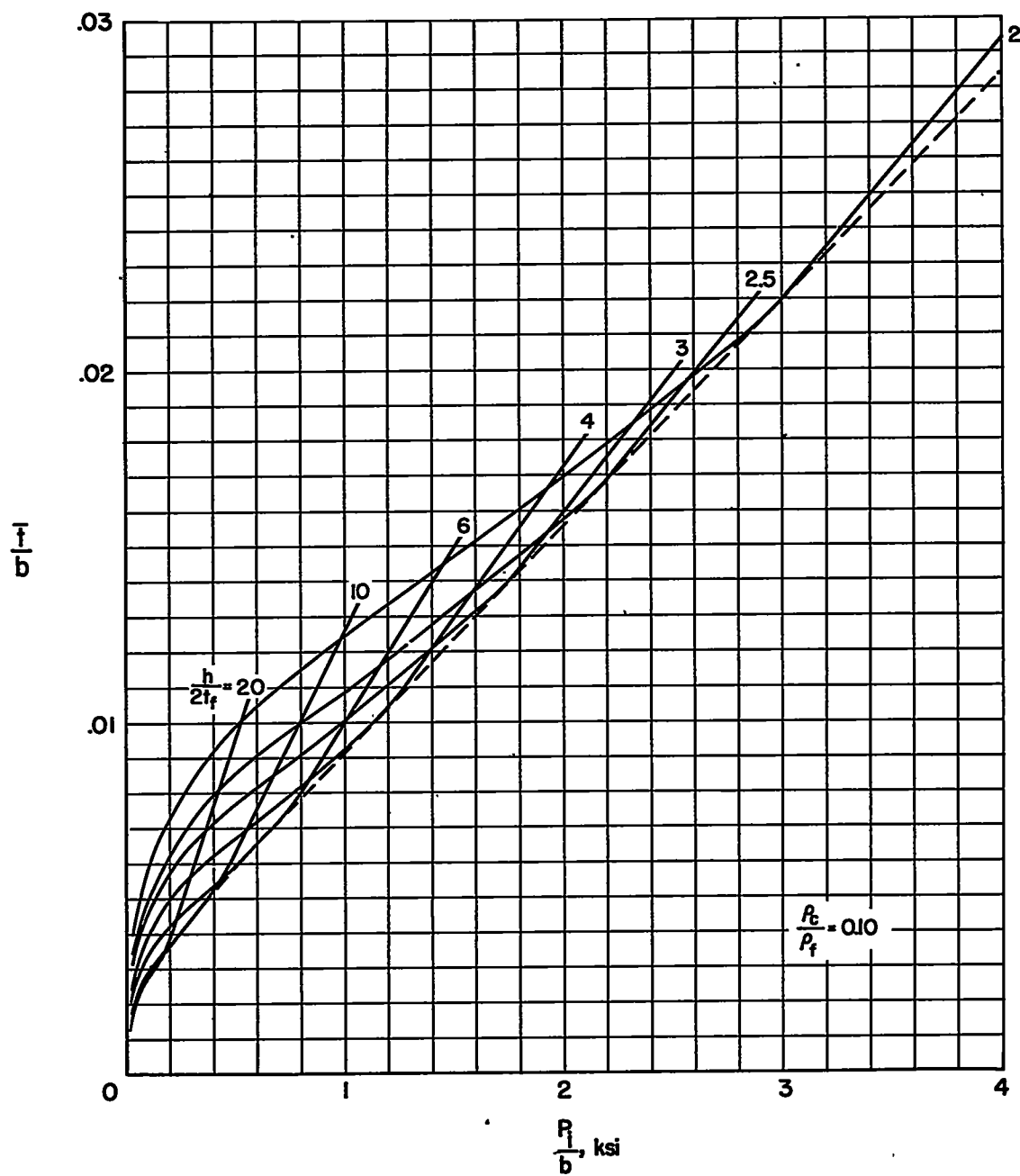
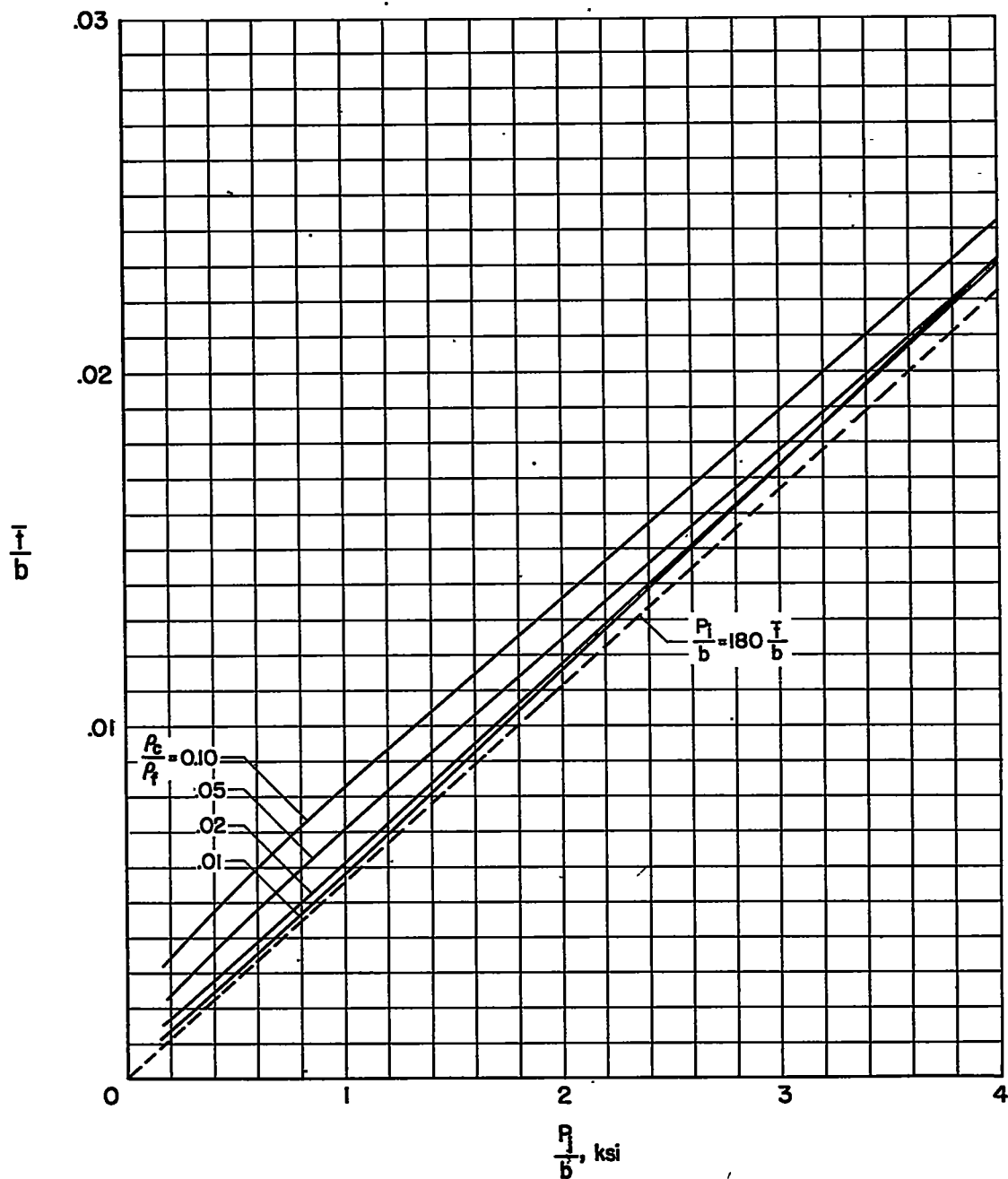Figure 3.- Continued.
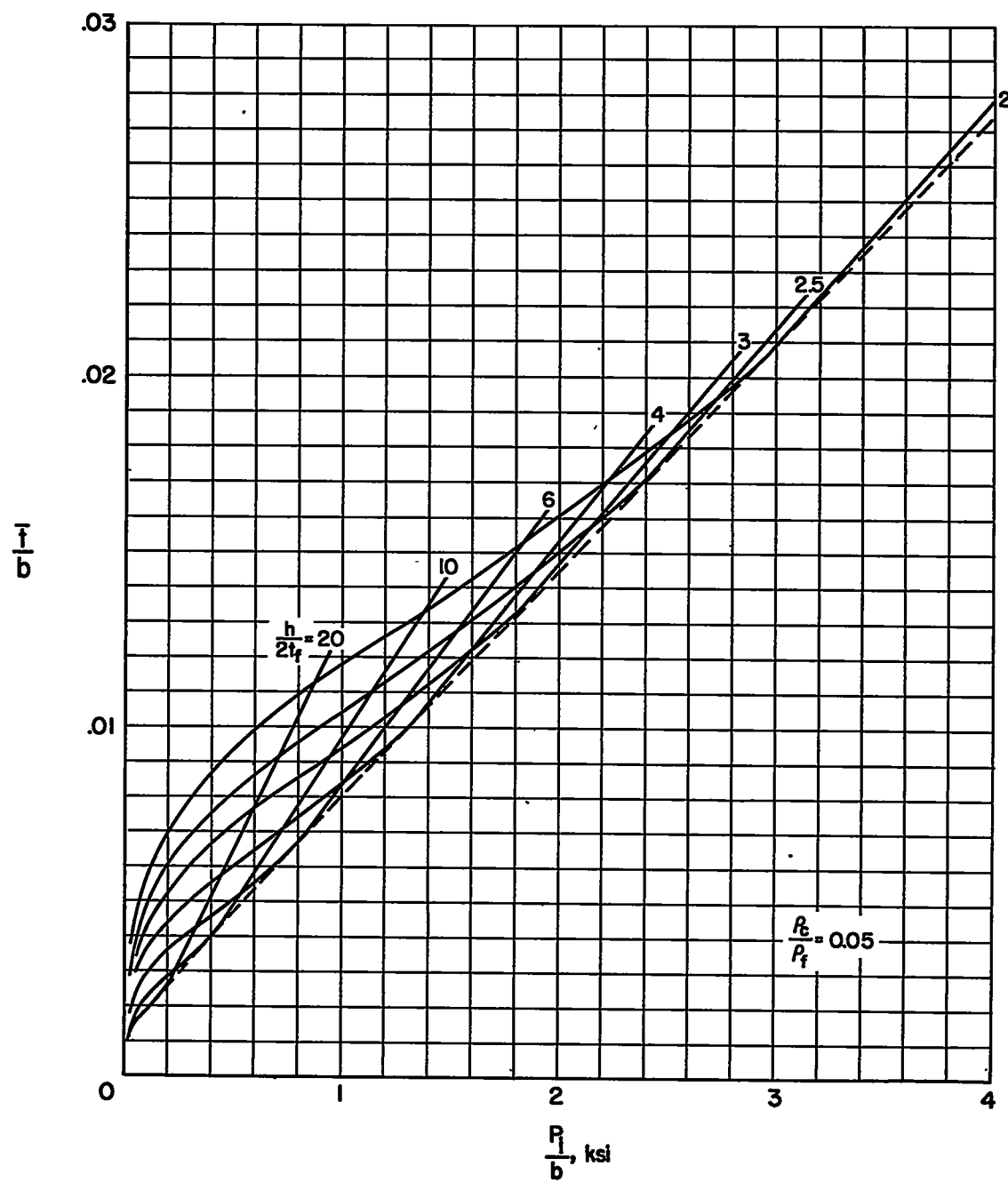
Figure 3.- Continued.

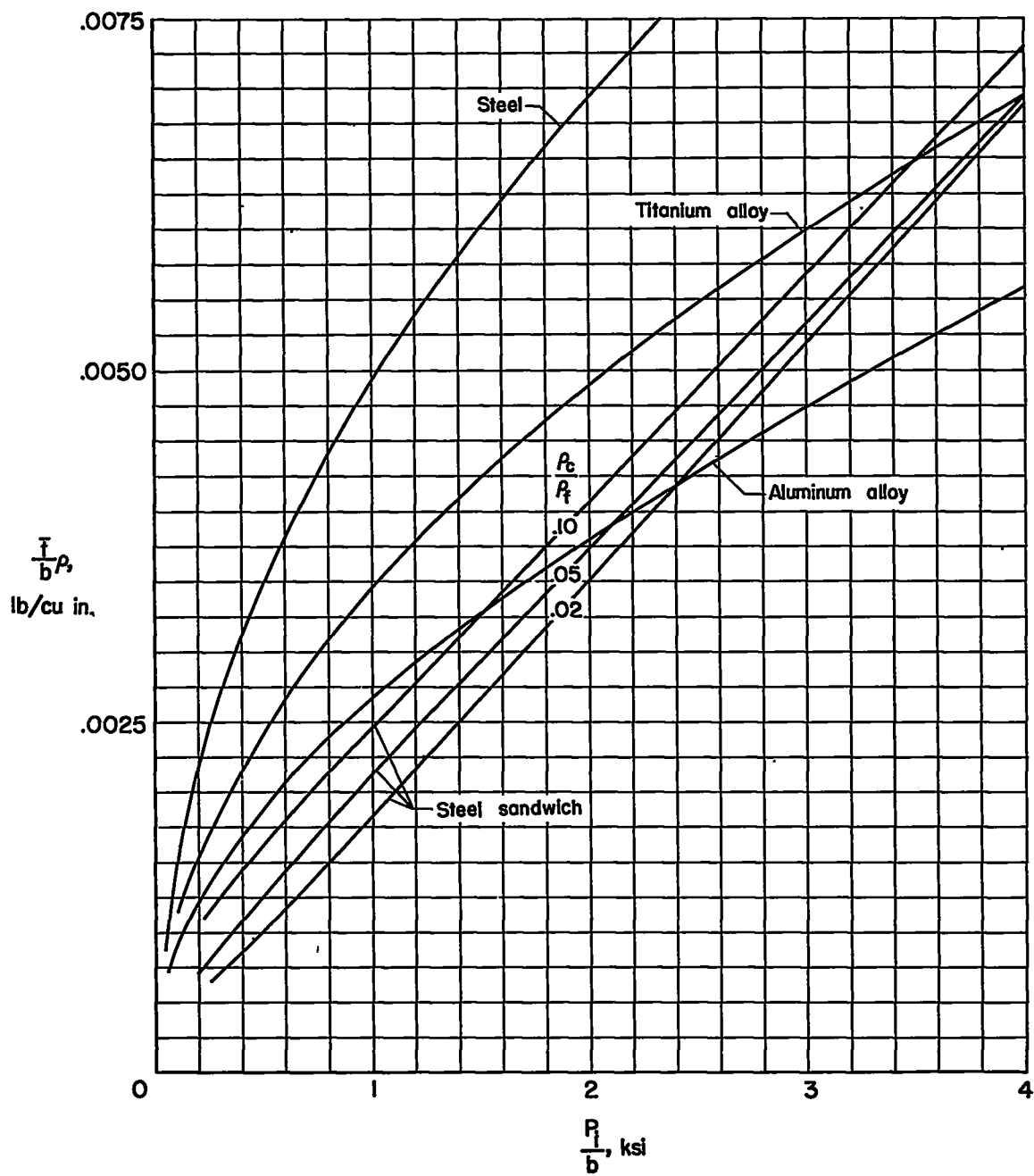Figure 3.- Concluded.

(a) Temperature, 80° F.

Figure 4.- Effect of core density on structural efficiency of high-strength, steel, cellular-core sandwich plates of optimum proportions.
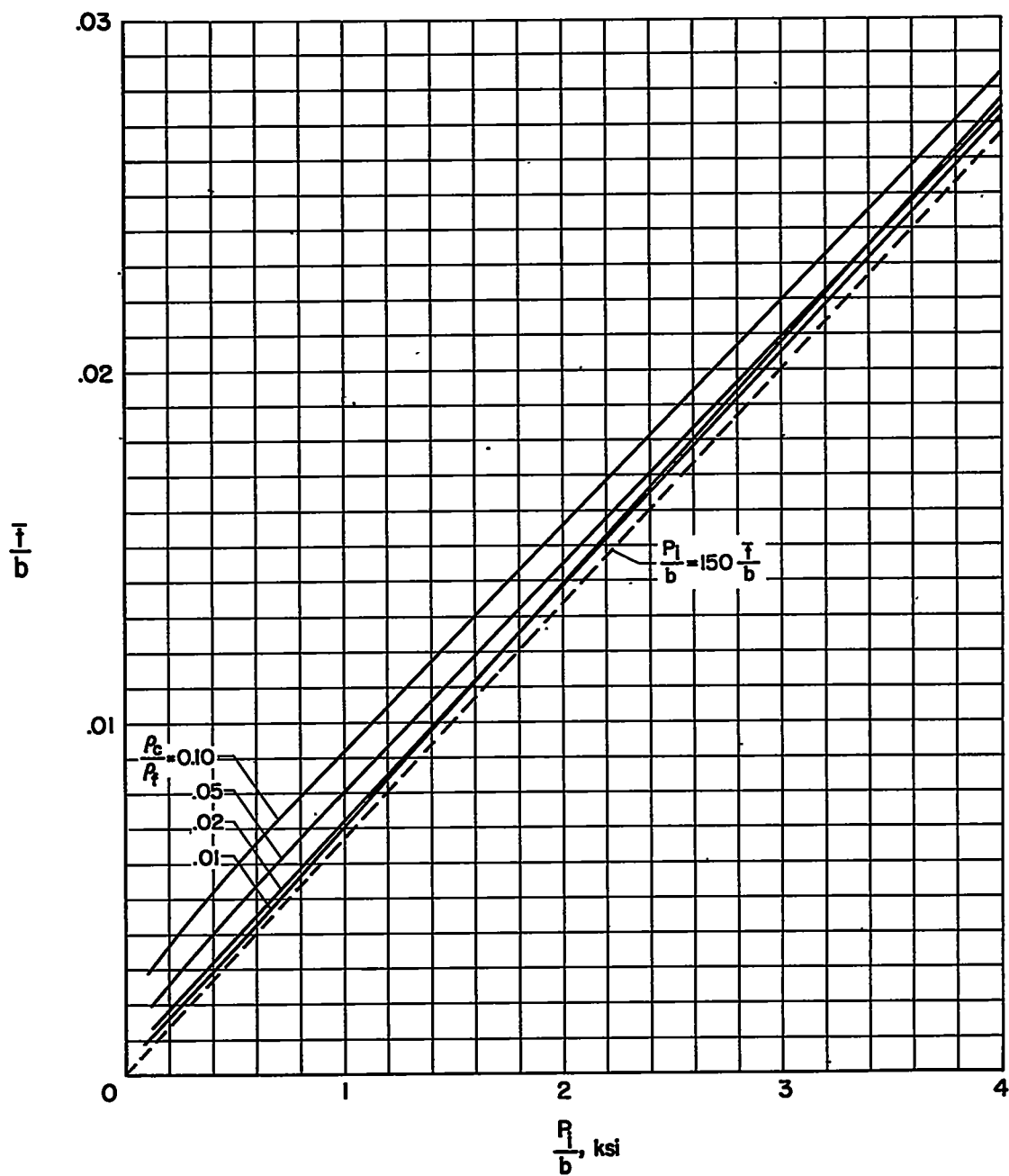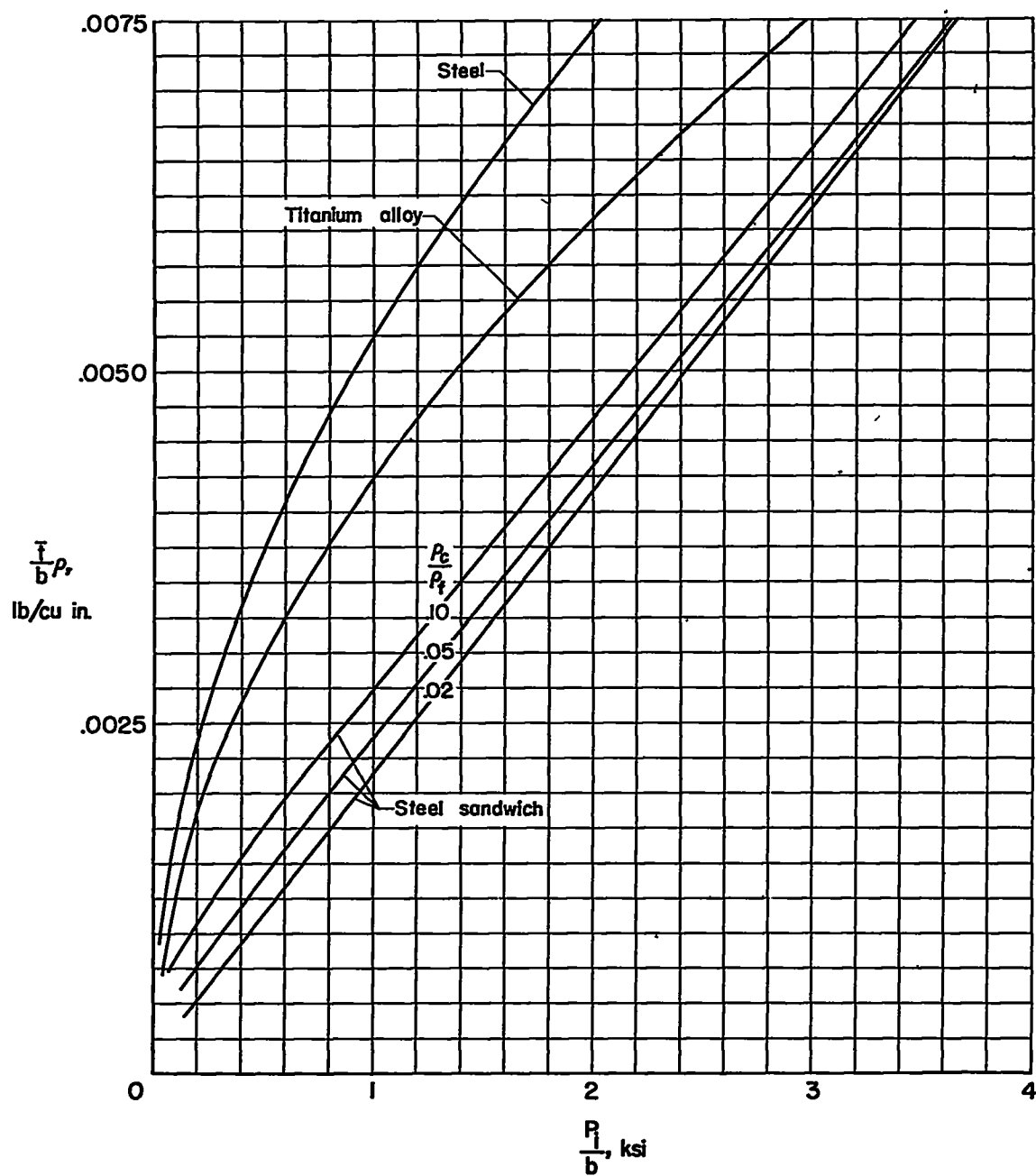
(b) Temperature, 600° F.

Figure 4.- Concluded.

(a) Temperature, 80° F.

Figure 5.- Comparison of optimum-proportioned, steel, cellular-core sandwich plates with solid plates of steel and aluminum and titanium alloys.

(b) Temperature, 600° F.

Figure 5.- Concluded.